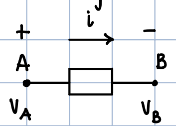
Voltage across a component:



Passive sign convention:

$V_{AB} = V_A - V_B$

Power: work / unit of time

Power across component:

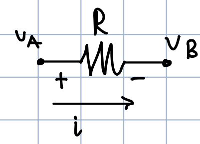
$\boxed{P = v \cdot i}$

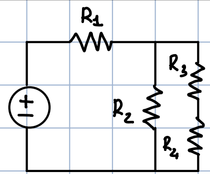
$P > 0$: consume

$P < 0$: generate

Ohm's Law: applies to a resistor

$\boxed{V = IR}$





R_3, R_4 series

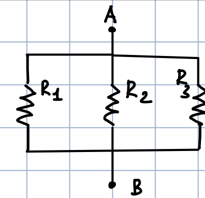
R_2, R_3, R_4 parallel

Resistors in series:

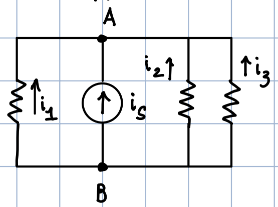
$\boxed{R_{eq} = R_1 + R_2 + R_3 + \dots + R_N}$

Resistors in parallel

$$\boxed{R_{eq} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \dots + \frac{1}{R_N}}}$$



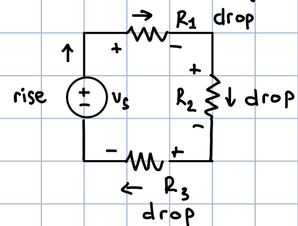
Kirchhoff's Current Law (KCL): Applies to nodes



$\boxed{\sum i_{in} = \sum i_{out}}$

KCL at A: $i_1 + i_2 + i_3 - i_s = 0$

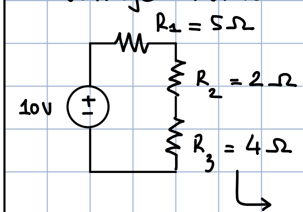
Kirchhoff's Voltage Law (KVL): Apply to mesh



$\boxed{\sum v_{rise} = \sum v_{drop}}$

$v_s - v_1 - v_2 - v_3 = 0$

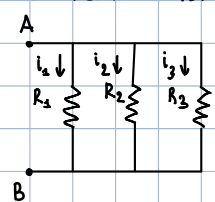
Voltage division: Series only



$$\boxed{v_i = \frac{R_i}{R_{eq}} v_{source}}$$

$$v_1 = \frac{5\ \Omega}{6\Omega} \cdot 10V = \frac{25}{3}\ V$$

Current division: Parallel only

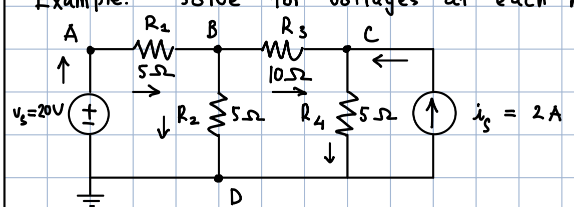


$$\boxed{i_i = \frac{R_{eq}}{R_i} \cdot i_{source}}$$

Nodal Voltage Method:

1. Identify number of nodes and name them.
2. Select ground node ⇒ choose one with most branches
3. Identify known node voltages.
4. Assume current direction ⇒ Apply KCL on known nodes
5. Solve using Ohm's Law.

Example: Solve for voltages at each node



1/ Select ground node at node D.

⇒ $v_D = 0V$; $v_A = 20V$ (20V voltage rise)

2/ KCL on nodes B and C:

$i_1 - i_2 - i_3 = 0$ (1) ; $i_3 + i_s - i_4 = 0$ (2)

$$i_1 = \frac{V_1}{R_1} = \frac{V_A - V_B}{5\Omega} = \frac{20 - V_B}{5} \ ; \ i_2 = \frac{V_2}{R_2} = \frac{V_B - V_D}{5\Omega} = \frac{V_B}{5}$$

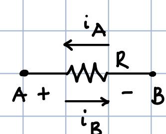
$$i_3 = \frac{V_3}{R_3} = \frac{V_B - V_C}{10} \ ; \ i_4 = \frac{V_4}{R_4} = \frac{V_C - V_D}{5} = \frac{V_C}{5}$$

Plug into (1) (2) to solve.

Mesh current method

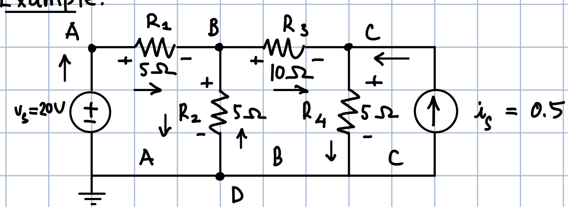
1. Identify and name meshes
2. Identify known current (current part of 1 mesh)
3. Assign clock-wise current flow in mesh and assign signs (+/−) to resistors following passive sign convention. (PSC)
4. Solve using KVL

Current as combination of meshes:



$\boxed{i_R = i_B - i_A \ (PSC)}$

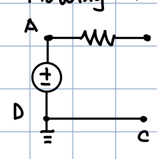
Example:



Mesh A: $v_s - v_1 - v_2 = 0 \Rightarrow 20 - 5i_1 - 5i_{2A} = 0.$

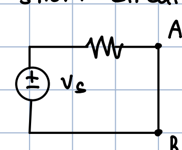
Mesh B: $v_{2B} - v_3 - v_{4B} = 0 \Rightarrow 5i_{2B} - 10i_3 - 5i_{4B} = 0$

Open circuit: No charge flowing ⇒ No current

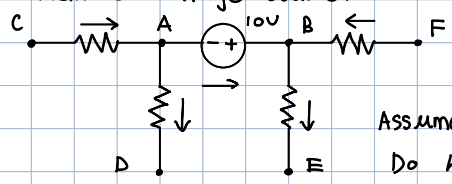


A, B replace by short circuit

Short circuit: Same Voltage

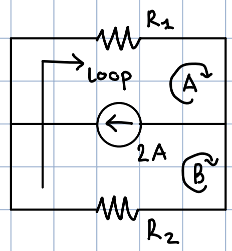


Supernode: Only exists when doing node voltage.
Non grounded voltage source
Not know current through voltage source ⇒
Remove voltage source.



1. Auxiliary equation:
$V_A - V_B = 10V$
Assume source short circuit
Do KCL on supernode

Supermesh: Current source between 2 adjacent meshes
Do Not know voltage across the current source
Remove branch with shared current source.



Auxiliary eq: $i_A - i_B = 2A$

KVL on the whole supermesh

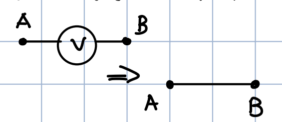
Principle of superposition:

- Calculate contribution (v_k ; i_k) of each source ⇒ solve for equivalence resistance (R_{TH} or R_N)
- Turn off all sources; leave 1 on

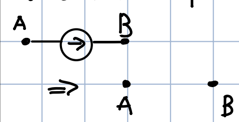
Define v or i to be found and solve for contribution of each source:

$i_{total} = i_{v\text{-source}} + i_{i\text{-source}}$

v-source: short



i-source: open

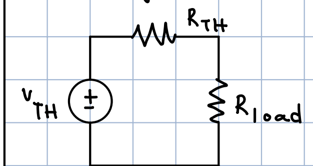


THÉVENIN CIRCUIT:

- Thévenin equivalent resistance R_{TH}:
  1. Remove load (open circ) and turn off all sources.
  2. Solve for equiva ent resistance

- Thévenin equivalent voltage:
  1. Remove the load, leaving the load terminals open.
  2. Define v_{TH} across the open terminals.
  3. Solve for v_{TH}.

- Thévenin equivalent circuit:
  voltage source + resistors in series.



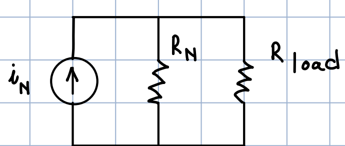
$$\boxed{I_N = \frac{V_{TH}}{R_{TH}}}$$

NORTON CIRCUIT

- Norton equivalent resistance R_N:
  1. Remove load (open circ) and turn off all sources.
  2. Solve for equivalent resistance

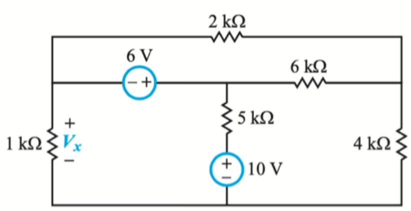
- Norton equivalent current:
  1. Replace load with short circuit.
  2. Define the short circuit's current as Norton's equivalent current i_N
  3. Solve for i_N

- Norton's equivalent circuit:

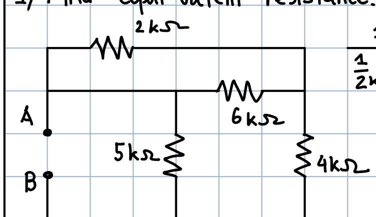


$$\boxed{V_{TH} = I_N \cdot R_{TH}}$$

Example: 1kΩ resistor is the load, find Norton and Thévenin:



1/ Find equivalent resistance:

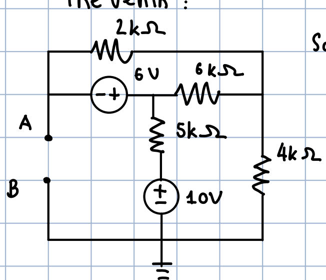


$$\frac{1}{\frac{1}{2k}+\frac{1}{6k}} = 1.5k\Omega$$

$1.5k\Omega + 4k\Omega = 5.5k\Omega$

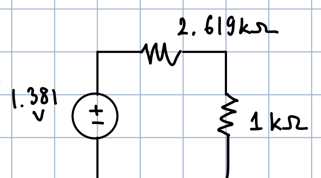
$$\frac{1}{\frac{1}{5k}+\frac{1}{5.5k}} = 2.619\,k\Omega$$

Thévenin:

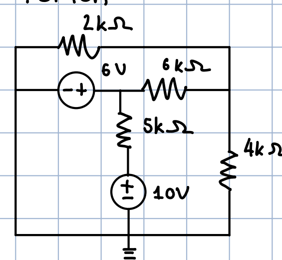


Solve using nodal voltage
Watch out for the supernode

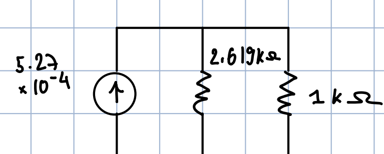
$v_A = 1.381\,V$
$v_B = 0V$
$v_{TH} = 1.381\,V$



Norton



Solve using nodal voltage
$i_N = 5.27 \times 10^{-4}\,A$



CAPACITORS: Unit: Capacitance [C]

Stores energy between electric field between 2 plates

PSC

$$\boxed{\begin{aligned} &C = Q/V \\ &i_c = i\,(dv/dt) \\ &v_c = \frac{1}{C}\int_0^t i_c(t)\,dt + v_0 \end{aligned}}$$

$\boxed{\text{Steady state: Open circuit}}$

In parallel: $C_{eq} = C_1 + C_2 + C_3 + \dots + C_N$

In series: $C_{eq} = \dfrac{1}{\frac{1}{C_1} + \frac{1}{C_2} + \dots + \frac{1}{C_N}}$

INDUCTORS: Unit: Henry [H]

Store energy in magnetic field in coil loop.

PSC

$$\boxed{\begin{aligned} &L = \Phi_B / I \\ &v_L = L\,(di_L/dt) \\ &i_L = \frac{1}{L}\int_0^t v_L(t)\,dt + i_0 \end{aligned}}$$

$\boxed{\text{Steady state: short circuit}}$

In series: $L_{eq} = L_1 + L_2 + \dots + L_N$

In parallel: $L_{eq} = \dfrac{1}{\frac{1}{L_1} + \frac{1}{L_2} + \dots + \frac{1}{L_N}}$

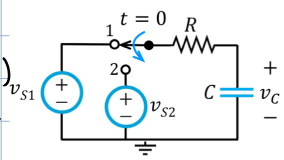
1st order circuit: contains 1 energy storage device: RL or RC circuit

Time constant τ: controls exponential rate of decay

$\boxed{\tau = RC \quad \tau = R/L}$

RC circuit: Transient respone

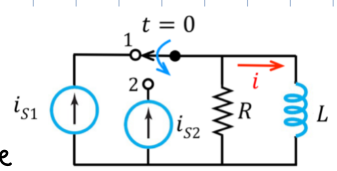
1. Calculate $v_c(0)$
2. Calculate steady state $v_c(\infty)$
3. Solve for transient response by finding Thévenin equivalent circuit with capacitor as the load.



$$\boxed{v_c(t) = v_c(\infty) + \big(v_c(0) - v_c(\infty)\big)\, e^{(-1/R_{TH}C)t}}$$

RL circuit: transient response

1. Calculate $i_L(0)$
2. Calculate steady $i_L(\infty)$
3. Solve for transient response by solving Norton circuit with L as a load.



$$\boxed{i_L(t) = i_L(\infty) + \big(i_L(0) - i_L(\infty)\big)\, e^{(-R_N/L)t}}$$

SINUSOIDAL SIGNAL: A: Amplitude ; φ: phase shift

ω: frequency

$\varphi < 0$: shift right

$\varphi >$: shift left

T: period ; f: frequency (Hz)

$\boxed{v_{AC} = A\cos(\omega t + \varphi)}$

$\boxed{\omega = 2\pi f}$ $\boxed{f = \frac{1}{T}}$

* Sine to Cosine signal:

Phase shift 90° to the right: $A\sin(\omega t - 90°) = A\cos(\omega t)$

COMPLEX NUMBER:

Rectangular: $\boxed{z = a + bj}$ → $\Re[z] = a$, $\Im[z] = b$

Polar: $\boxed{z = m\angle\theta = me^{\theta j}}$ → Magnitude $[z] = m$, Phase $[z] = \theta$

Rec → Polar: $m = \sqrt{a^2 + b^2}$; $\theta = \tan^{-1}(b/a)$

Polar → Rec: $a = m\cos\theta$; $b = m\sin\theta$

PHASOR DOMAIN: transform from time domain to phasor domain to avoid sine and ODE

$$\boxed{v(t) = A\cos(\omega t + \varphi) \Rightarrow v(\omega j) = Ae^{\varphi j}}$$

Impedence: Ratio between phasor voltage and phasor current

$\boxed{Z = \frac{V}{I}}$

Resistors	$Z_R = R$
Capacitors	$Z_C = \dfrac{1}{j\omega C}$
Inductors	$Z_L = j\omega L$

AC circuit Analysis:

1. Write sources in cosine form.
2. Convert to phasor domain
3. Use source frequency to write impedences for components ; redraw in phasor form.
4. Use DC circuit methods ⇒ convert back to time domain

Phasor domain equivalent circuit:

Find Norton and Thévenin equivalent circuits in phasor domain.

AC Power:

Given: $v(t) = v_m \cos(\omega t + \varphi)$

$i(t) = i_m \cos(\omega t + \varphi)$

$$\boxed{P = \frac{1}{T_0}\int_0^{T_0} v(t)i(t)\,dt = \frac{V_m I_m}{2}\cos(\emptyset_V - \emptyset_I) = \frac{V_m I_m}{2}\cos(\emptyset_z)}$$

⇒ Average amount of work done or energy transferred per unit time

Average Power for Components: Polar Form only!

Resistors: $Z_R = Re^{0j}$ ⇒ $\boxed{P_R = \frac{V_m \cdot I_m}{2}}$

Capacitor: $\boxed{P_c = 0}$

Inductors: $\boxed{P_L = 0}$

Ohm's Law in phasor form:

$\boxed{V = IZ}$ $\boxed{V = V_m e^{\emptyset_v j}}$

$\boxed{I = I_m e^{\emptyset_I j}}$ $\boxed{Z = \frac{V_m}{I_m} e^{(\emptyset_v - \emptyset_I)j}}$

$Z_m = \frac{V_m}{I_m}$ $\emptyset_z = \emptyset_V - \emptyset_I$

COMPLEX POWER:

Complex = Real P + reactive P

$$\boxed{S = \frac{VI^*}{2} = P + jQ}$$

$*$: indicates complex conjugate

$I = a + bj$

$\Rightarrow I^* = a - bj$

Given: $\mathbf{V} = V_m e^{\emptyset_v j}$; $\mathbf{I} = I_m e^{\emptyset_I j}$

$$\Rightarrow S = \frac{V_m e^{\emptyset_v j} \cdot I_m e^{-\emptyset_I j}}{2} = \frac{V_m I_m}{2} e^{(\emptyset_v - \emptyset_I)j}$$

$$S = S_m \cos(\emptyset_S) + S_m \sin(\emptyset_S)j = \frac{V_m I_m}{2}\cos(\emptyset_v - \emptyset_I) + \frac{V_m I_m}{2}\sin(\emptyset_v - \emptyset_I)j$$

Real Average Power:

$$\boxed{P_{av} = \Re e[S] = \frac{V_m I_m}{2}\cos(\emptyset_v - \emptyset_I)}$$

Reactive power: $\boxed{Q = \Im m[S] = \frac{V_m I_m}{2}\sin(\emptyset_v - \emptyset_I)}$

Apparent power: $\boxed{S = \sqrt{P_{av}^2 + Q^2} = \frac{V_m I_m}{2}}$

Power factor: $\boxed{pf = \frac{P_{av}}{S} = \cos(\emptyset_v - \emptyset_i)}$

$*$: complex conjugate

m: magnitude

$X \cdot X^* = X_m^2$

$X_m \angle \theta_X \cdot X_m \angle -\theta_X$

$X_m^2 \angle 0$

TRANSFER FUNCTION:

describes output response to an input as function of angular frequency ω

$H = \frac{\text{output}}{\text{input}}$

$\boxed{\text{Voltage gain}: \quad H = \frac{V_{out}}{V_{in}}}$

Transfer impedance: $H_Z = \frac{V_{out}}{I_{in}}$

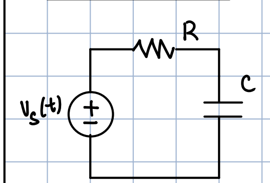
Current gain: $H_Z = \frac{I_{out}}{I_{in}}$

Transfer Admittance: $H_Y = \frac{I_{out}}{V_{in}}$

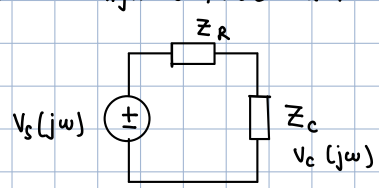
V_{in} and I_{in} are independent voltage / current source ; V_{out}, I_{in} are freely chosen.

Frequency response: measure how circuit responds to sinusoidal inputs of arbitrary frequency.

Low pass Filter : Signals with high ω filtered out.



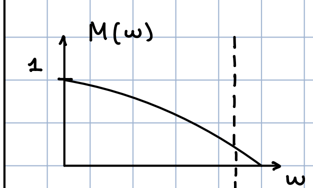
Phasor →



$V_c(j\omega) = \frac{1}{1 + j\omega CR} \cdot V_s(j\omega)$ ⇒ voltage division

$$\boxed{H = \frac{V_c(j\omega)}{V_s(j\omega)} = \frac{1}{1 + j\omega CR}}$$

$$M(\omega) = \frac{1}{\sqrt{1 + (\omega CR^2)}}$$

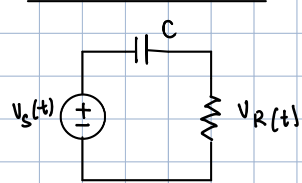


As $\omega \to 0$, $M(\omega) \to 1$

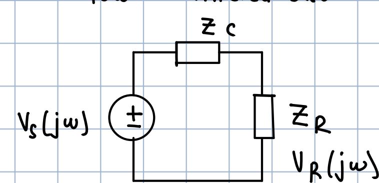
$\omega \to \infty$, $M(\omega) \to 0$

Cut-off : $\omega = \frac{1}{CR}$

HIGH PASS FILTER: Signals with low ω filtered out



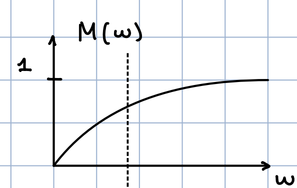
→



$$V_R(j\omega) = \frac{Z_R}{Z_R + Z_c} \cdot V_s(j\omega) = \frac{R}{R + 1/j\omega C} \cdot V_s(j\omega)$$

$$\boxed{H = \frac{V_R(j\omega)}{V_s(j\omega)} = \frac{j\omega CR}{1 + j\omega CR}}$$

$$M(\omega) = \frac{\omega CR}{\sqrt{1 + (\omega CR^2)}}$$



As $\omega \to 0$, $M(\omega) \to 0$

$\omega \to \infty$, $M(\omega) \to 1$

BODE PLOTS: Frequency response plot displayed in logarithmic scale. Horizontal axis: frequency ↑
Unit: decade 10^N. Vertical axis: amplitude or phase unit: dB. $20\log_{10}(M(\omega))$

$H(j\omega) = \frac{\text{output}}{\text{input}} \Rightarrow M(\omega) \angle \phi(\omega)$

Cut-off frequency: $\boxed{P_{out} < \frac{1}{2} P_{in}}$ $\boxed{\frac{V_{out}}{V_{in}} < \frac{1}{\sqrt{2}}}$ $\boxed{\left|\frac{P_{out}}{P_{in}}\right|_{dB} = 20\log_{10}\left(\frac{V_{out}}{V_{in}}\right)}$

$20\log_{10}\left(\frac{1}{\sqrt{2}}\right) = -3dB \Rightarrow$ filtering begins

Pole: values make denominator $= 0$ → $\frac{1}{1+j\omega R}$
Zero: values make numerator $= 0$

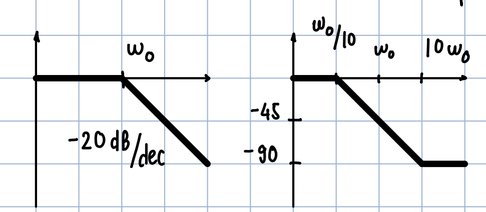
Table 9-2: Bode straight-line approximations for magnitude and phase.

Factor	Bode Magnitude	Bode Phase
Constant K	$20\log K$; 0 dB	$\pm 180°$ if $K<0$; $0°$ if $K>0$
Zero @ Origin $(j\omega)^N$	0 dB; 1; slope $= 20N$ dB/decade	$(90N)°$; $0°$
Pole @ Origin $(j\omega)^{-N}$	0 dB; 1; slope $= -20N$ dB/decade	$0°$; $(-90N)°$
Simple Zero $(1+j\omega/\omega_c)^N$	0 dB; ω_c; slope $= 20N$ dB/decade	$0°$; $0.1\omega_c$; ω_c; $10\omega_c$; $(90N)°$
Simple Pole $\left(\frac{1}{1+j\omega/\omega_c}\right)^N$	0 dB; ω_c; slope $= -20N$ dB/decade	$0°$; $0.1\omega_c$; ω_c; $10\omega_c$; $(-90N)°$
Quadratic Zero $[1+j2\xi\omega/\omega_c+(j\omega/\omega_c)^2]^N$	0 dB; ω_c; slope $= 40N$ dB/decade	$0°$; $0.1\omega_c$; ω_c; $10\omega_c$; $(180N)°$
Quadratic Pole $\frac{1}{[1+j2\xi\omega/\omega_c+(j\omega/\omega_c)^2]^N}$	0 dB; ω_c; slope $= -40N$ dB/decade	$0°$; $0.1\omega_c$; ω_c; $10\omega_c$; $(-180N)°$

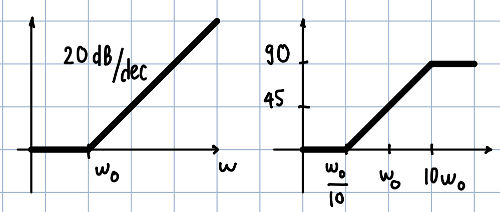
Real pole: $H(s) = \frac{1}{1+j\omega/\omega_0}$

$= \frac{1}{\sqrt{1+(\omega/\omega_0)^2}} \angle -\arctan\left(\frac{\omega}{\omega_0}\right)$

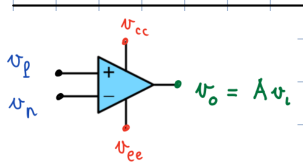
Cut-off: $x = -\omega_0 \Rightarrow$ critical freq



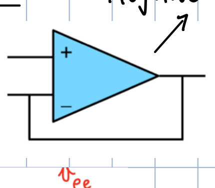
Real zero: $H(s) = 1 + \frac{j\omega}{\omega_0}$



OPERATIONAL AMPLIFIERS:



Negative feedback



Properties:

- High Gain: $A = 10^5 - 10^6$
- Differential input: $\boxed{v_{in} = v_p - v_n}$
- Linear operating range:
- Saturated by $+v_{cc}$ and $-v_{ee}$
- Usually $\boxed{v_{ee} = -v_{cc}}$

$$\boxed{\begin{aligned}\text{Output} &= \text{Input} \cdot \text{Gain} \\ &= (V_p - V_n) \cdot \text{Gain}\end{aligned}}$$

AMPLIFIER BEHAVIOR MODEL:

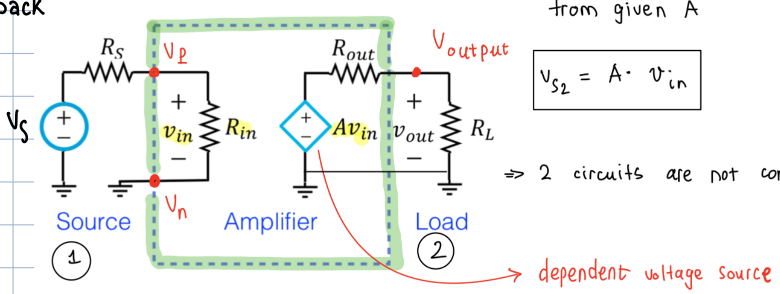
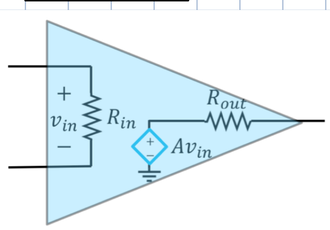


Figure out v_{in} first from given A

$\boxed{v_{S_2} = A \cdot v_{in}}$

$\Rightarrow$ 2 circuits are not connected.

IDEAL MODEL:



$\boxed{v_{in} = \frac{R_{in}}{R_{in}+R_s} v_s}$

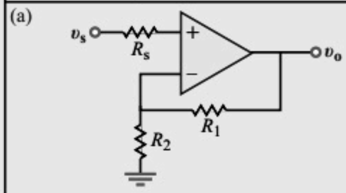
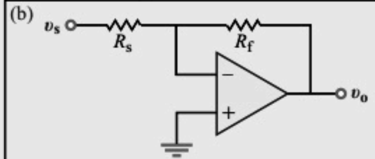
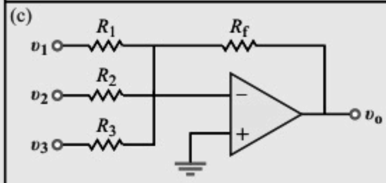
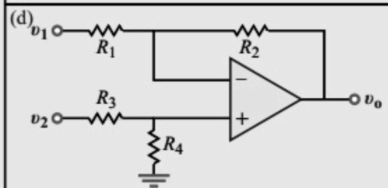
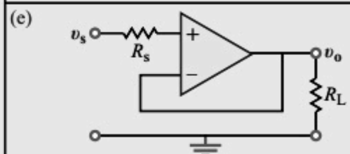
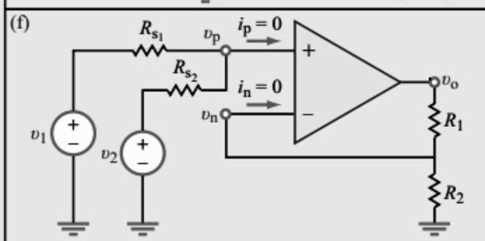
$\boxed{v_{out} = v_{S_2} \frac{R_L}{R_L+R_{out}}}$

Assumptions of ideal models:

1. $i_p = i_n = 0A \Rightarrow$ open circuit

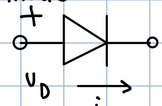
2. $v_p = v_n$ (voltage is the same, different node).

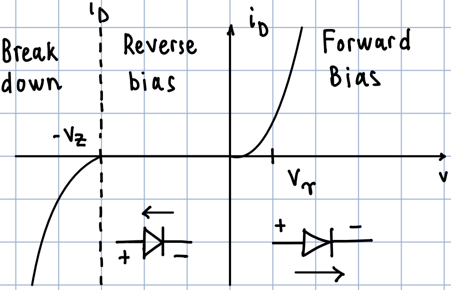
Table 4-3: Summary of op-amp circuits.

Op-Amp Circuit	Block Diagram
(a) 	$v_s \rightarrow \boxed{G = \frac{R_1 + R_2}{R_2}} \rightarrow v_o = Gv_s$ Noninverting Amp (v_o independent of R_s)
(b) 	$v_s \rightarrow \boxed{G = -\frac{R_f}{R_s}} \rightarrow v_o = Gv_s$ Inverting Amp
(c) 	$v_1 \rightarrow \boxed{G_1 = -R_f/R_1}$, $v_2 \rightarrow \boxed{G_2 = -R_f/R_2}$, $v_3 \rightarrow \boxed{G_3 = -R_f/R_3}$ $\rightarrow (+) \rightarrow v_o = G_1v_1 + G_2v_2 + G_3v_3$ Inverting Summing Amp
(d) 	$v_1 \rightarrow \boxed{G_1 = -\frac{R_2}{R_1}}$, $v_2 \rightarrow \boxed{G_2 = \left(\frac{R_1+R_2}{R_1}\right)\left(\frac{R_4}{R_3+R_4}\right)}$ $\rightarrow (+) \rightarrow v_o = G_1v_1 + G_2v_2$ Subtracting Amp
(e) 	$v_s \rightarrow \boxed{G = 1} \rightarrow v_o = v_s$ Voltage Follower / Buffer (v_o independent of R_s and R_L)
(f) 	$v_1 \rightarrow \boxed{G_1 = \left(\frac{R_1+R_2}{R_2}\right)\left(\frac{R_{s_2}}{R_{s_1}+R_{s_2}}\right)}$, $v_2 \rightarrow \boxed{G_2 = \left(\frac{R_1+R_2}{R_2}\right)\left(\frac{R_{s_1}}{R_{s_1}+R_{s_2}}\right)}$ $\rightarrow (+) \rightarrow v_o = G_1v_1 + G_2v_2$ Noninverting Summing Amp

DIODE: p-n junction
p = holes ; n = electrons

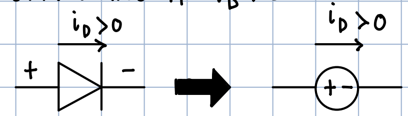
Anode + Cathode −



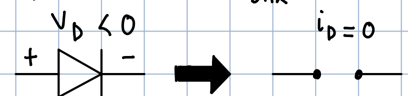


Simple Diode Model:

1/ Forward Mode: replaced with 0.7V source if $i_D > 0$

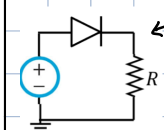


2/ Reverse Mode: replaced with open circuit if $-v_{BRK} < v_D < 0.7$

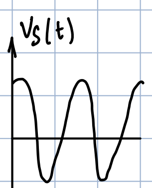


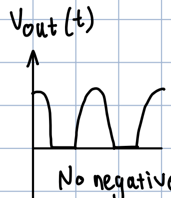
Solving problems: Assume all diodes in Reverse mode first. Check conditions. Change accordingly

RECTIFIER DIODE: AC source



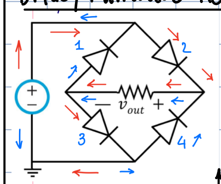
← Halfway rectifier
Check each mode, condition → plot



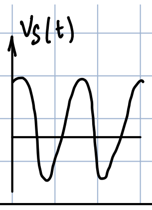


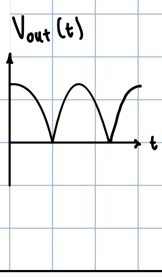
No negative

Bride/Fullwave Rectifier:

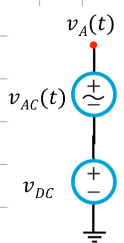


Lost magnitude
flipped signal



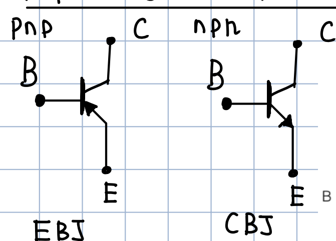


Mixed AC/DC signal:

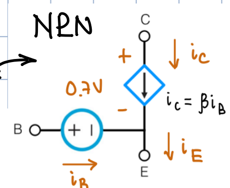


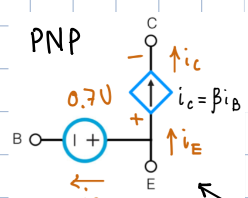
Output: DC is vertical shift, same frequency.

Bipolar Junction Transistor (BJT):



Active mode:





$i_B > 0$
$V_{CE} \geq 0.3V$
$V_{EC} \geq 0.3$

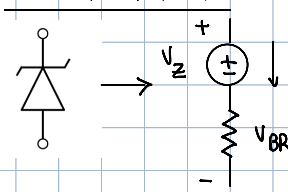
Zener Diode.

3 main states:

1. For $v_D \geq v_{ON}$: conventional forward-biased diode. ⇒ 0.7
2. For $v_Z < v_D < v_{ON}$: open circuit. v_{BRK} $v_D < 0.7V$
3. For $v_D \leq v_Z$: Zener breakdown, voltage across diode is $-v_Z$.

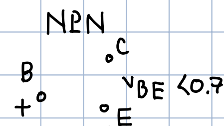
Breakdown Mode:

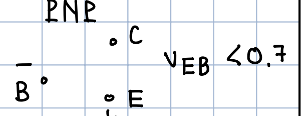
$$\boxed{v_Z = v_{BRK} + i \cdot r_Z}$$



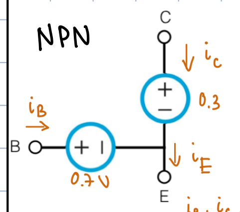
Given like this:
$\begin{cases} v_Z = 5V @ 1mA \\ r_Z = 25\Omega \end{cases}$

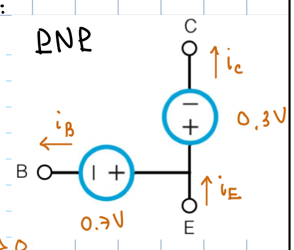
Cut off mode:





Saturation mode:





$i_B, i_C, i_E \geq 0$
$i_C < \beta i_B$

Cut-off → Active → Saturation